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Interpreting Interactions

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1 Introduction

Regression models are often used to explore associations between different variances, sometimes including interactions. Unfortunately, interactions are sometimes hard to interpret. Here we explain the interpretation of three different kinds of interactions

- 1. nominal (sometimes called categorical, or binary if there are only two categories) by nominal;
- 2. nominal by continuous; and
- 3. continuous by continuous.

Code examples in STATA and R using the birthweight dataset are provided.

2 Birthweight data

The birthweight data set birthwt can be found in the package MASS in R.

In STATA, the dataset 1bw can be loaded from the web directly.

It should be noted that the R and STATA versions of the dataset are not exactly the same, and therefore the results shown below are slightly different. See Appendix A for a comparison.

3 Linear regression

3.1 Nominal by nominal

Without interaction With only main effects, we assume that the mean difference between categories of one variable is the same, regardless of the value of the 2nd variable, and vice versa.

With interaction Including an interaction term, we assume that the mean difference between categories of one variable differs according to the 2nd variable, and vice versa.

Interpretation of Interaction Coefficient The interaction term gives additional difference in means for non-reference levels of the two categorical variables.

Interpretation The reference category for smoke is non-smoking mothers, and for nonwhite is white mothers. Babies of smokers have on average -601.9g lower birthweights than non-smokers. Babies of non-white mothers have -604.2g lower birthweights than those of whites. However, the association with birthweight is not as strong as expected in non-white smokers, as they have on average 419.5g higher birthweights than would be expected considering the main effects only.

Interpretation for each group

Non-smoking, white mothers This is the reference group, with an average birthweight given by the intercept: 3428.7g.

Smoking, white mothers White mothers who smoke have babies with on average -601.9g lower birthweights than white mothers who do not smoke.

Non-smoking, non-white mothers Non-white mothers who do not smoke have babies with on average - 604.2g lower birthweights than white mothers who do not smoke.

Smoking, non-white mothers Non-white mothers who do smoke have babies with on average -601.6 + -604.2 + 419.5 = -786.3g lower birthweights than white mothers who do not smoke.

```
m1 <- lm(bwt ~ smoke * nonwhite, data = birthwt)
summary(m1)
##
## Call:
## lm(formula = bwt ~ smoke * nonwhite, data = birthwt)
## Residuals:
   Min 1Q Median 3Q
## -2407.75 -416.85 31.25 483.25 1561.25
## Coefficients:
                             Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                             3428.7 102.7 33.378 < 2e-16 ***
## smokesmoker
                             -601.9
                                        139.6 -4.312 0.00002624 ***
## nonwhitenonwhite
                              -604.2
                                        130.7 -4.622 0.00000712 ***
                               419.5
## smokesmoker:nonwhitenonwhite
                                        217.1
                                               1.932 0.0548.
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 681.4 on 185 degrees of freedom
## Multiple R-squared: 0.1408, Adjusted R-squared: 0.1268
## F-statistic: 10.1 on 3 and 185 DF, p-value: 0.000003393
```

. regress bwt i.smoke##i.nonwhite, noheader							
bwt	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]	
smoke							
smoker	-601.3654	139.5211	-4.31	0.000	-876.6224	-326.1084	
1.nonwhite	-605.4401	130.685	-4.63	0.000	-863.2646	-347.6156	
smoke#							
nonwhite							
smoker#1	420.1464	216.9997	1.94	0.054	-7.965728	848.2586	
_cons	3428.75	102.6848	33.39	0.000	3226.166	3631.334	

3.2 Nominal by continuous

Without interaction With only main effects, we assume that the slope of y over the continuous variable, z is the same regardless of the category of the nominal variable, z = 0 or z = 1. That is, the regression lines for each group in z are parallel.

With interaction Including an interaction term, we assume that the slope of y over x differs according to z = 0 or z = 1. The regression lines for each group in z no longer are assumed to be parallel.

Interpretation of Interaction Coefficient The interaction term gives additional change in slope of y over x for the non-reference level of the nominal variable, z = 1. The slopes are given by:

$$z = 0$$
: $\hat{\beta}_x$
 $z = 1$: $\hat{\beta}_x + \hat{\beta}_{x:z}$

Interpretation For non-smokers, average birthweight increases by 27.7g per year of age of the mother. For smokers, the average birthweight actually decreases by -18.8g (27.73 + -46.57) per year of age of the mother.

```
m2 <- lm(bwt ~ smoke * age, data = birthwt)
summary(m2)

##

## Call:
## lm(formula = bwt ~ smoke * age, data = birthwt)
##

## Residuals:
## Min 1Q Median 3Q Max
## -2189.27 -458.46 51.46 527.26 1521.39

##

## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2406.06 292.19 8.235 3.18e-14 ***
## smokesmoker 798.17 484.34 1.648 0.1011
## age 27.73 12.15 2.283 0.0236 *
## smokesmoker:age -46.57 20.45 -2.278 0.0239 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 709.3 on 185 degrees of freedom

## Multiple R-squared: 0.06909, Adjusted R-squared: 0.054

## F-statistic: 4.577 on 3 and 185 DF, p-value: 0.004068
```

```
. regress bwt i.smoke##c.age, noheader

bwt | Coef. Std. Err. t P>|t| [95% Conf. Interval]

smoke |
smoker | 797.9369  484.3249  1.65  0.101  -157.5731  1753.447
age | 27.60058  12.14868  2.27  0.024  3.632806  51.56835

smoke#c.age |
smoker | -46.51558  20.44641  -2.28  0.024  -86.85368  -6.177479

__cons | 2408.383  292.1796  8.24  0.000  1831.951  2984.815
```

Tip Note that the main effect of smoking here gives the mean difference between smokers and non-smokers for age = 0. It may be easier to interpret models with nominal by continuous interactions if you first center the continuous variable (at mean, median or other relevant value).

```
median(birthwt$age)

## [1] 23

birthwt$agec <- birthwt$age - 23
m2c <- lm(bwt ~ smoke * agec, data = birthwt)
summary(m2c)$coef

## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3043.87967 66.34054 45.882648 5.136217e-103
## smokesmoker -272.97916 105.82868 -2.579444 1.067228e-02
## agec 27.73138 12.14910 2.282587 2.359245e-02
## smokesmoker:agec -46.57191 20.44711 -2.277677 2.388962e-02
```

agec 27.60058 12.14868 2.27 0.024 3.632806 51.56835 smoke#c.agec smoker -46.51558 20.44641 -2.28 0.024 -86.85368 -6.177479	smoker	-271.9214	105.825	-2.57	0.011	-480.7004	-63.14238	
	agec	27.60058	12.14868	2.27	0.024	3.632806	51.56835	
	smoke#c.agec							
		-46.51558	20.44641	-2.28	0.024	-86.85368	-6.177479	
_cons 3043.196 66.33825 45.87 0.000 2912.32 3174.073	_cons	3043.196	66.33825	45.87	0.000	2912.32	3174.073	

3.3 Continuous by continuous

Without interaction With only main effects, we assume that the slope of y over the continuous variable x1 is the same regardless of x2 and vice versa.

With interaction Including an interaction term, we assume that the slope of y over the continuous variable x1 differs with respect to x2, and vice versa.

Interpretation of Interaction Coefficient The interaction term gives the change in slope of *y* over *x*1 for each unit of *x*2, and the change in slope of *y* over *x*2 for each unit of *x*1. The actual slopes are given by:

```
slope over x1: \hat{\beta}_{x1} + x_2 \hat{\beta}_{x1:x2}
slope over x2: \hat{\beta}_{x2} + x_1 \hat{\beta}_{x1:x2}
```

Interpretation Average birthweight increases by on average 11.7g for every year of the mother's age, and 4.4g for each pound of the mother's weight. Increasing age and weight of the mother make these associations slight less pronounced (-0.3g per year of age and pound).

Tip Unless x1 = 0 and x2 = 0 are meaningful in your dataset, you may end up with strange values for the intercept or other main effect estimates. If this happens, try centering continuous variables. Don't forget that this will change the calculation of the predicted values:

$$\hat{y} = \hat{\beta}_{(Intercept)} + \hat{\beta}_{agec}(age-23) + \hat{\beta}_{lwtc}(lwt-121) + \hat{\beta}_{agec:lwtc}(age-23)(lwt-121)$$

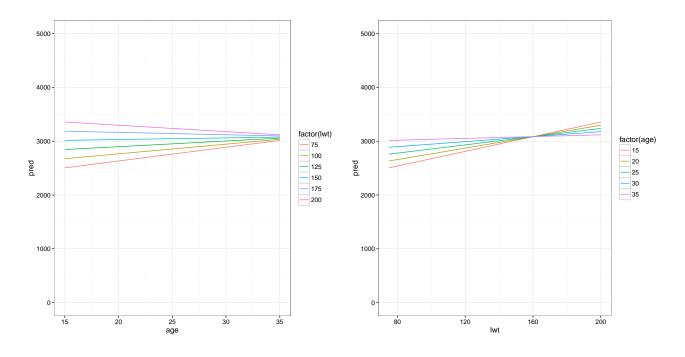
```
median(birthwt$lwt)
## [1] 121
birthwt$lwtc <- birthwt$lwt - 121
m3 <- lm(bwt ~ agec * lwtc, data = birthwt)
summary(m3)
##
## Call:
## lm(formula = bwt ~ agec * lwtc, data = birthwt)
## Residuals:
## Min 1Q Median 3Q
## -2258.87 -477.29 16.28 512.40 1824.01
##
## Coefficients:
   Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 2912.1115 54.8888 53.055 <2e-16 ***
## agec 11.7363 10.8076 1.086 0.279
```

```
## lwtc     4.4237     1.7645     2.507     0.013 *
## agec:lwtc     -0.2992     0.3227     -0.927     0.355
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 719.4 on 185 degrees of freedom
## Multiple R-squared: 0.04229, Adjusted R-squared: 0.02676
## F-statistic: 2.723 on 3 and 185 DF, p-value: 0.04569
```

```
. centile lwt
                                   -- Binom. Interp. --
  Variable | Obs Percentile Centile [95% Conf. Interval]
_______
          189 50
    lwt
                          121
                                     120
                                             128
. gen lwtc = lwt - 121
. regress bwt c.agec##c.lwtc, noheader
     bwt | Coef. Std. Err. t P>|t| [95% Conf. Interval]
------
    agec | 11.57163 10.80576 1.07 0.286 -9.746736 32.88999
   lwtc | 4.425356 1.764444 2.51 0.013
                                    .9443383 7.906374
   c.agec#
   c.lwtc | -.2953255 .3226567 -0.92 0.361 -.9318852 .3412342
    _cons | 2911.685 54.88113 53.05 0.000 2803.411 3019.958
```

Tip Graph the predicted values in order to make sense of continuous by continuous interactions.

```
nd <- expand.grid(agec = seq(15, 35, 5) - 23, lwtc = seq(75, 200, 25) - 121)
nd$pred <- predict(m3, newdata = nd)
nd$age <- nd$agec + 23
nd$lwt <- nd$lwtc + 121
qplot(age, pred, data = nd, color = factor(lwt), geom = "line") + ylim(0, 5000)
qplot(lwt, pred, data = nd, color = factor(age), geom = "line") + ylim(0, 5000)</pre>
```



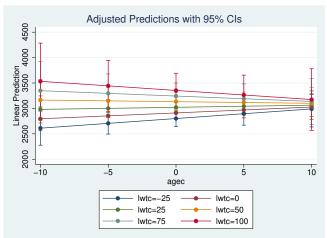
. quietly: margins, at(agec = (-10(5)10) lwtc = (-25(25)100))

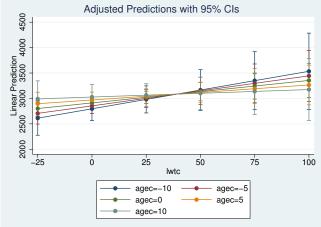
. marginsplot

Variables that uniquely identify margins: agec lwtc

. marginsplot, xdim(lwtc)

Variables that uniquely identify margins: agec lwtc





4 Logistic regression

The interpretations given in this section apply equally to

- logistic regression for binary outcomes ($e^{\hat{\beta}}$ = odds ratio (OR)),
- poisson regression for count outcomes $(e^{\hat{\beta}} = \text{incidence rate ratio (IRR)}),$
- Cox proportional hazards regression for survival outcomes $(e^{\hat{\beta}} = \text{hazard ratio (HR)})$,
- and other regression models where relevant coefficients are interpreted as $e^{\hat{\beta}}$, not $\hat{\beta}$.

4.1 Nominal by nominal

Without interaction With only main effects, we assume that the odds ratio comparing categories of one variable is the same, regardless of the value of the 2nd variable, and vice versa.

With interaction Including an interaction term, we assume that the odds ratio comparing categories of one variable differs according to the 2nd variable, and vice versa. An OR < 1 for the interaction, indicates the association is less strong than expected when considering only the main effects, while OR > 1 indicates the association is stronger than expected.

Interpretation of Interaction Coefficient The interaction term gives multiplicative effect of non-reference levels of the two categorical variables.

For nominal by nominal interactions, we examine the effects of two covariates simultaneously by multiplying the odds ratios. To see the effect of covariates x1 and x2, we multiply $e^{\hat{\beta}_{x1}}$ with $e^{\hat{\beta}_{x2}}$ to get $e^{\hat{\beta}_{x1}}e^{\hat{\beta}_{x2}}=e^{\hat{\beta}_{x1}+\hat{\beta}_{x2}}$. (Note that we can either a) first add the coefficients and then exponentiate, or b) first exponentiate to get odds ratios, and then multiply.) With interaction, we calculate the odds ratio as follows:

$$OR_{x1,x2} = e^{\hat{\beta}_{x1}} e^{\hat{\beta}_{x2}} e^{\hat{\beta}_{x1:x2}}.$$

Interpretation In this example, we look at the odds of having birthweight less than 2.5kg. Smokers have 5.76 higher odds of having a baby with low birthweight compared to non-smokers. Similarly, nonwhite mothers have a 5.43 higher odds of having a baby with low birthweight compared to white mothers. Nonwhite mothers who smoke however have a 10 times higher odds of having a baby with low birthweight than white mothers who do not smoke.

STATA Tip Note the use of the coeflegend option to find out what the coefficients are called, in case you want to use them in calculations.

```
. logistic low i.smoke##i.nonwhite
                                        Number of obs = 189
LR chi2(3) = 16.97
Logistic regression
                                                         0.0007
                                         Prob > chi2
Log likelihood = -108.84968
                                        Pseudo R2
                                                     =
                                                          0.0723
______
      low | Odds Ratio Std. Err.
                                  z \qquad P > |z|
                                              [95% Conf. Interval]
------
    smoke

    smoker
    5.757576
    3.444621
    2.93
    0.003
    1.782321
    18.59916

    1.nonwhite
    5.434783
    3.153762
    2.92
    0.004
    1.742756
    16.94837

     smoke#
  nonwhite
  smoker#1 .3195789 .2478524 -1.47 0.141
                                               .069891
                                                        1.461286
     _cons | .1 .0524404 -4.39 0.000 .0357788 .2794949
. logistic, coeflegend
                                         Number of obs = 189
LR chi2(3) = 16.97
Logistic regression
                                        LR chi2(3) =
                                        Prob > chi2 = 0.0007
Log likelihood = -108.84968
                                        Pseudo R2
                                                          0.0723
------
      low | Odds Ratio Legend
     smoke
   smoker | 5.757576 _b[1.smoke]
 1.nonwhite | 5.434783 _b[1.nonwhite]
     smoke#
  nonwhite
  smoker#1 | .3195789 _b[1.smoke#1.nonwhite]
     _cons | .1 _b[_cons]
                             . di \exp(b[1.smoke]) * \exp(b[1.nonwhite]) * \exp(b[1.smoke#1.nonwhite])
10
. * or equivalently:
. di exp(_b[1.smoke] + _b[1.nonwhite] + _b[1.smoke#1.nonwhite])
```

4.2 Nominal by continuous

Without interaction With only main effects, we assume that the odds ratio increases the same amount per unit of the continuous variable, x, is the same regardless of the category of the nominal variable, z = 0 or z = 1.

With interaction Including an interaction term, we assume that the change in odds ratio over the continuous variable differs according the value of z

Interpretation of Interaction Coefficient The interaction term gives additional change in odds for the non-reference level of the nominal variable, z = 1. The ORs are given by:

$$z = 0$$
: e^{β_x}
 $z = 1$: $e^{\beta_x}e^{\beta_{x:z}}$

Interpretation In this example, the odds of having a baby with low birthweight decreases by a factor of 0.92 per every year of the mother's age if the mother doesn't smoke, and by a factor of 0.92 * 1.08 = 0.99 for every year if she does smoke.

```
m5 <- glm(low ~ smoke * agec, data = birthwt, family = binomial)

cbind("OR" = exp(coef(m5)), exp(confint(m5)))

## Waiting for profiling to be done...

## OR 2.5 % 97.5 %

## (Intercept) 0.3324575 0.2115932 0.5048132

## smokesmoker 2.0492797 1.0889112 3.8894061

## agec 0.9204617 0.8382765 1.0009474

## smokesmoker:agec 1.0758199 0.9474313 1.2256759
```

```
. logistic low i.smoke##c.agec, coef
                                    Number of obs =
Logistic regression
                                                      189
                                    LR chi2(3) =
                                                    8.66
                                    Prob > chi2
                                              =
                                                  0.0342
Log likelihood = -113.00535
                                    Pseudo R2
                                                    0.0369
             Coef. Std. Err.
                             z P>|z|
                                         [95% Conf. Interval]
smoke
   smoker .7174884 .3237495 2.22 0.027
                                        .0829511 1.352026
     agec | -.0828798 .0449925 -1.84 0.065 -.1710635
                                                  .0053039
smoke#c.agec |
   smoker | .0730831 .0653439 1.12 0.263
                                         -.0549886
                                                 .2011548
     _cons | -1.101243 .2206746 -4.99 0.000
                                        -1.533758 -.668729
```

4.3 Continuous by continuous

Without interaction With only main effects, we assume that the change in OR over the continuous variable x1 is the same regardless of x2 and vice versa.

With interaction Including an interaction term, we assume that the change in OR over the continuous variable x1 differs with respect to x2, and vice versa.

Interpretation of Interaction Coefficient The interaction term gives the change in OR over x1 for each unit of x2, and the change in slope of y over x2 for each unit of x1. The actual slopes are given by:

```
slope over x1: e^{\beta_{x_1} + x_2} \beta_{x_1:x_2} slope over x2: e^{\beta_{x_2} + x_1} \beta_{x_1:x_2}
```

Interpretation The odds ratios considering an interaction between age and weight are *very* slightly lower (99.9% of the odds ratio considering only main effects [99.7 - 1.002%] per year of age and pound in weight), but this difference is not statistically significant.

Tip Plotting predicted odds ratios or probabilities for these models will make the models easier to understand.

```
m6 <- glm(low ~ agec * lwtc, data = birthwt, family = binomial)
cbind("OR" = exp(coef(m6)), exp(confint(m6)))

## Waiting for profiling to be done...

## OR 2.5 % 97.5 %

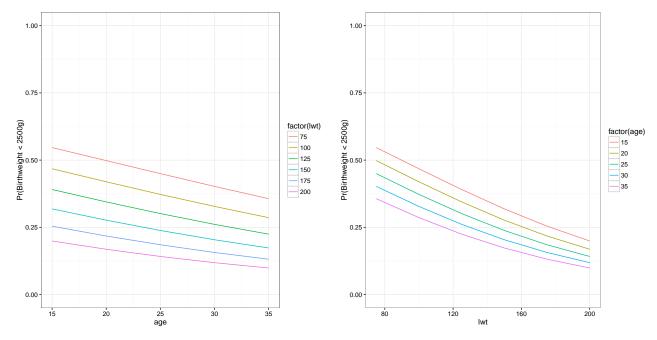
## (Intercept) 0.4907450 0.3535745 0.6731492

## agec 0.9611041 0.8986580 1.0242901

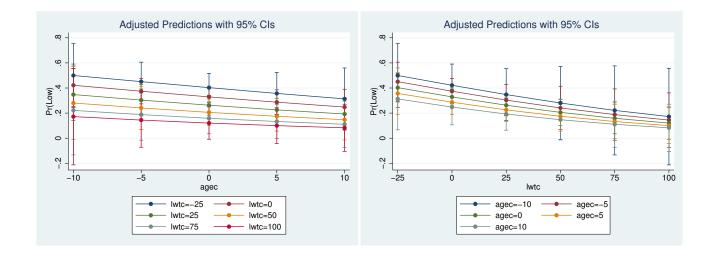
## lwtc 0.9873092 0.9745634 0.9987438

## agec:lwtc 0.9999823 0.9974573 1.0022256
```

```
nd <- expand.grid(agec = seq(15, 35, 5) - 23, lwtc = seq(75, 200, 25) - 121)
nd$pred <- predict(m6, newdata = nd, type = "response")
nd$age <- nd$agec + 23
nd$lwt <- nd$lwtc + 121
qplot(age, pred, data = nd, color = factor(lwt), geom = "line") +
   ylim(0, 1) + ylab("Pr(Birthweight < 2500g)")
qplot(lwt, pred, data = nd, color = factor(age), geom = "line") +
   ylim(0, 1) + ylab("Pr(Birthweight < 2500g)")</pre>
```



```
. logistic low c.agec##c.lwtc, coef
Logistic regression
                                            Number of obs =
                                                                 189
                                            LR chi2(3)
                                                                 7.53
                                            Prob > chi2
                                                               0.0567
Log likelihood = -113.56918
                                            Pseudo R2
                                                                0.0321
       low | Coef. Std. Err. z P>|z| [95% Conf. Interval]
      agec | -.0396806 .033211 -1.19 0.232
                                                   -.104773
                                                             .0254118
      lwtc | -.0127505 .0062141 -2.05 0.040 -.0249298 -.0005711
     c.agec#
     c.lwtc | -.0000202 .0011979 -0.02 0.987 -.002368
                                                            .0023275
      _cons | -.7117894 .1638335 -4.34 0.000 -1.032897 -.3906816
. quietly: margins, at (agec = (-10(5)10) | wtc = (-25(25)100))
. marginsplot
 Variables that uniquely identify margins: agec lwtc
. marginsplot, xdim(lwtc)
 Variables that uniquely identify margins: agec lwtc
```



Versions

1.0 Original version

R version and packages used to generate this report

R version: R version 3.4.0 (2017-04-21)

Base packages: stats, graphics, grDevices, utils, datasets, methods, base

Other packages: MASS, ggplot2, knitr

This document was generated on 2017-06-22 at 13:35.

A Comparison of R and STATA Datasets

There are a few small differences in lwt and bwt between the two versions of the dataset we use here, which led to slight differences in the model results.

```
data.r <- birthwt[, c("age", "lwt", "bwt", "race", "smoke")]</pre>
data.stata <- read.csv("lbw_stata.csv")</pre>
summary(data.r$age - data.stata$age)
    Min. 1st Qu. Median Mean 3rd Qu. Max.
              0
                             0
summary(data.r$lwt - data.stata$lwt)
      Min. 1st Qu.
                       Median
                                 Mean 3rd Qu.
## -1.000000 0.000000 0.000000 -0.005291 0.000000 0.000000
data.r[data.r$lwt - data.stata$lwt != 0, ]
## age lwt bwt race
## 76 20 105 2450 other non-smoker
data.stata[data.r$lwt - data.stata$lwt != 0, ]
## age lwt bwt race
## 182 20 106 2450 other nonsmoker
summary(data.r$bwt - data.stata$bwt)
    Min. 1st Qu. Median Mean 3rd Qu.
## -14.0000 0.0000 0.0000 0.3016 0.0000 69.0000
data.r[data.r$bwt - data.stata$bwt != 0, "bwt"]
## [1] 2751 3062 3062 3544 2410
data.stata[data.r$bwt - data.stata$bwt != 0, "bwt"]
## [1] 2750 3076 3076 3475 2395
table(data.r$race, data.stata$race, useNA = "ifany")
##
##
        black other white
    white 0 0 96 black 26 0 0
##
   _ack 26
##
            0 67
##
table(data.r$smoke, data.stata$smoke, useNA = "ifany")
##
##
             nonsmoker smoker
## non-smoker 115 0
## smoker 0 74
```